

Constraints on Disformal Couplings from the CMB Temperature Evolution

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(Dated: March 20, 2013)

Certain modified gravity theories predict the existence of an additional, non-conformally coupled scalar field. A disformal coupling of the field to the Cosmic Microwave Background (CMB) is shown to affect the evolution of the energy density in the radiation fluid. Therefore, measurements of the CMB temperature at various redshifts can be used to constrain these disformal couplings. Such measurements strongly support the predictions of General Relativity, that the CMB temperature evolution with redshift is linear. For both exponential and power law potentials for the scalar field we find an excluded range for the strength of this coupling, characterised by an energy scale M , to be $\text{few} \times 10^{-6} \text{ eV} < M < \text{few} \times 10^{-3} \text{ eV}$. For certain values of M , we find that the effective disformal coupling to radiation becomes singular.

PACS numbers:

The observational evidence of an accelerated expansion of the Universe has inspired cosmologists to study theories in which scalar field(s) or other matter forms play the role of dark energy. The simplest explanation, a cosmological constant, works very well in the sense that it fits the observations, but its value is hard to explain within theories beyond the standard model of particle physics. Among the simplest extensions to a cosmological constant are quintessence models, in which a scalar field, usually assumed to be uncoupled to matter, rolls down a potential. This happens slowly enough so that the pressure is negative and the scalar field can cause the expansion of the universe to speed up. However, these models have their own problems. Why should the field be uncoupled? Why should the potential be rather flat and how can radiative corrections to the potential be kept small? How can these models be embedded in theories beyond the standard model? We refer to [1] and [2] for detailed discussions about models of dark energy.

More complicated models have therefore been considered and extensions of Einstein's General Theory of Relativity explored (see e.g. [3]). A rather broad class of models has attracted a lot of attention recently, the so-called scalar-tensor theories. The metric to which matter and/or radiation couples is given by

$$\tilde{g}_{\mu\nu} = C(\phi)g_{\mu\nu} + D(\phi)\partial_\mu\phi\partial_\nu\phi. \quad (1)$$

The function $C(\phi)$ is called the conformal coupling and $D(\phi)$ the disformal coupling. Different matter species might couple differently to the field ϕ (therefore, the functions $C(\phi)$ and $D(\phi)$ are not the same for different species), but in this paper, we will not consider this case.

A metric which determines the geodesics of matter is required to be consistent with both causality and the

weak equivalence principle. This must hold for any extension of General Relativity. As shown by Bekenstein, in [4], the metric given in eq. (1) is consistent with those constraints.

These scalar-tensor theories with a generalised metric address some but not all the questions raised above, yet they can be phenomenologically very rich and interesting in their own right. Conformal couplings of a scalar field to matter (either baryonic or dark matter) have been studied extensively, but only recently has the attention shifted to consider disformal couplings as well [5]-[12]. Disformal couplings are motivated in several field theories, such as those originating in brane worlds or theories of massive gravity. In massive gravity theories, ϕ describes the helicity-zero mode of the massive spin-2 particle and D is related to the mass of the graviton. In this paper we study, for the first time, the cosmological consequences of radiation disformally coupled to a scalar field. Unlike in theories with conformal couplings, disformal couplings affect radiation, even though the trace of the energy momentum tensor vanishes. We will be focusing on disformal couplings only.

The action we are considering is of scalar-tensor form, namely

$$\mathcal{S} = \int \sqrt{-g} d^4x \left[\frac{\mathcal{R}}{16\pi G} - \frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V(\phi) \right] + S_\gamma(\tilde{g}_{\mu\nu}) + S_{\text{mat}}(g_{\mu\nu}), \quad (2)$$

in which \mathcal{R} is the Ricci scalar, S_{mat} and S_γ are the actions for matter and radiation fluids, respectively, and the two metrics are related by a disformal transformation:

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \frac{1}{M^4}\partial_\mu\phi\partial_\nu\phi. \quad (3)$$

M is a mass scale describing the disformal coupling of radiation to the scalar degree of freedom ϕ . Note that the disformal coupling goes to zero if $M \rightarrow \infty$ and we recover General Relativity. M could be a function of ϕ itself, but we are not discussing this possibility further

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in the paper. We expect our conclusions to be valid for more general situations in which M is not constant. Note that C has been set to 1 without loss of generality, due to the vanishing trace of radiation's stress-energy tensor.

Photons travel on the light cone given by the disformal metric and therefore $d\tilde{s}^2 = \tilde{g}_{\mu\nu}dx^\mu dx^\nu = 0$. Following [13], we find that the redshift z is related to the scale factor via

$$1+z = \frac{a_0}{a} \sqrt{\frac{M^4 - \dot{\phi}^2}{M^4 - \dot{\phi}_0^2}}, \quad (4)$$

where the subscript 0 means values are taken at the present time. The additional factor stems from the fact that \tilde{g}_{00} depends on the time-derivative of the scalar field and varies with time itself. Note that in limit $M \rightarrow \infty$ we recover the standard result of General Relativity, as expected.

The field equations can easily be obtained from the action above [11] (see also [9] for discussions of disformal couplings to matter). The scalar field equation is given by

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi - \frac{dV}{d\phi} + Q = 0, \quad (5)$$

with

$$Q = -\nabla_\nu \left(\frac{1}{M^4} \phi_{,\mu} T_{(\gamma)}^{\mu\nu} \right) \quad (6)$$

and $T^{\mu\nu}$ being the energy-momentum tensor of radiation, which is consequently not conserved:

$$\nabla_\mu T_{(\gamma)\nu}^\mu = Q\phi_{,\nu}. \quad (7)$$

The energy-momentum tensor of matter, on the other hand, is conserved:

$$\nabla_\mu T_{(\text{mat})\nu}^\mu = 0. \quad (8)$$

Considering a homogeneous and isotropic universe, for which the metric is given by

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j, \quad (9)$$

Q then reduces to

$$Q = \frac{4H\dot{\phi} + V'}{M^4 + (\rho_\gamma - \dot{\phi}^2)} \rho_\gamma \quad (10)$$

While the equations for all matter species have the same form as in General Relativity, the equation for radiation is modified, because of the presence of the coupling. The equation for ρ_γ , the energy density of radiation, reads

$$\dot{\rho}_\gamma + 4H\rho_\gamma = -Q\dot{\phi}. \quad (11)$$

The expression for Q depends on the cosmic time derivative of the scalar field, $\dot{\phi}$, and therefore Q will depend on the potential energy $V(\phi)$. In this paper we will

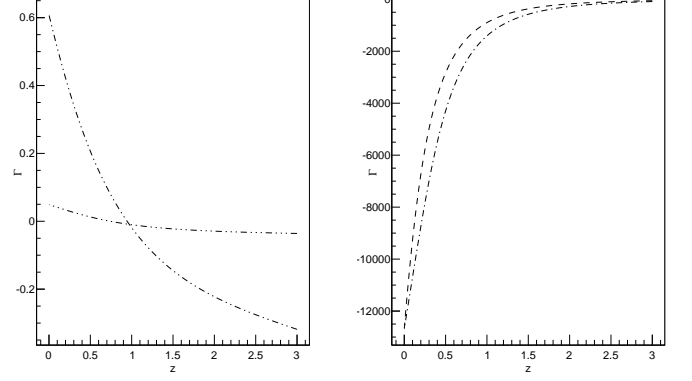


FIG. 1: Evolution of the effective coupling, Γ , between the scalar field ϕ and radiation with redshift, for the exponential potential. Left: Low M regime, $M^4 \ll (\rho_{\gamma,0} - \dot{\phi}_0^2)$. Right: Large M regime, $M^4 \gg (\rho_{\gamma,0} - \dot{\phi}_0^2)$. Each line represents a different M : dashed line is $M = 1.53 \times 10^{-5}$ eV, dashed line with one dot, $M = 2.17 \times 10^{-5}$ eV, dashed line with two dots, $M = 3.10 \times 10^{-3}$ eV, and dashed line with three dots is $M = 3.07 \times 10^{-3}$ eV.

study two potentials, which differ in slope. The first is a simple exponential potential

$$V = V_0 e^{-\lambda\phi}, \quad (12)$$

where we choose $\lambda = 1.0 M_{\text{Pl}}^{-1}$, and V_0 such that the scalar field plays the role of dark energy today. The second potential is of the inverse power law form

$$V = \frac{\mathcal{M}^{4+\alpha}}{\phi^\alpha}, \quad (13)$$

where we choose $\alpha = 6$. We tune \mathcal{M} to get the correct cosmology today. This potential will be steeper than the exponential form of eq. (12), and will therefore lead to a different equation of state for dark energy.

We begin the discussion of our results with the simpler, well behaved solutions of the exponential potential. We integrate the equations for matter, radiation (eq. (11)) and the Klein-Gordon equation (eq. (5)) such that we obtain a present day universe with density parameters $\Omega_{\text{m},0} = 0.3$ (cold dark matter and baryons), $\Omega_{\phi,0} = 0.7$ (dark energy) [14] and $T_0 = 2.725$ K (CMB temperature) [15]. To reproduce these cosmological observables, we choose to fix the field's initial value, $\phi_{\text{ini}} = 1.5 M_{\text{Pl}}$, but vary V_0 . To estimate the influence of the disformal coupling on the evolution of radiation, we introduce a new parameter

$$\Gamma = \frac{Q}{\rho_\gamma}, \quad (14)$$

which measures the strength of the disformal coupling to radiation. We have defined this quantity in analogy to

the effective coupling of pressureless matter to a scalar field. The evolution of Γ as a function of redshift is shown in Fig. 1.

First to note in the behaviour of Γ is that it does not stay finite for certain values of M . That Γ will go singular at some point in time if $(M^4 + \rho_\gamma) \simeq \dot{\phi}^2$ is evident from the denominator of eq. (10). This is clearly unphysical, and when Γ blows up, no realistic solutions for ρ_γ exist for that range of M . We find numerically that the range for where there is no acceptable cosmology is given by

$$4.33 \times 10^{-5} \text{ eV} \lesssim M \lesssim 1.37 \times 10^{-3} \text{ eV} . \quad (15)$$

All signatures from our disformal couplings occur for M values either side of this resonance, and become quickly suppressed as $M \rightarrow 0, \infty$. This singularity in Γ separates the solutions into two distinct regimes, which we recognise as the following: A low M regime, $M^4 \ll (\rho_{\gamma,0} - \dot{\phi}_0^2)$, and large M regime, $M^4 \gg (\rho_{\gamma,0} - \dot{\phi}_0^2)$. The 0 subscripts indicate present day values.

For low M , (shown on the left in Fig. 1), the field derivative and the radiation energy density are the dominant contribution of the denominator in the expression for Q (see eq. (10)). For $z < 3$, Γ is negative, implying that the radiation fluid gains energy from the scalar field in that period, as one would expect from eq. (11). For large M (shown on the right in Fig. 1), the denominator in eq. (10) is dominated by M^4 . Γ is initially negative but becomes positive, and therefore the radiation fluid begins to lose energy to the scalar field.

To investigate some observational consequences of the theory, we now study the evolution of the CMB temperature as a function of redshift, which has been measured in the range $0 \leq z \leq 3$ by various authors [15–18]. In order to relate the energy density ρ_γ to the temperature T , we follow [16, 19] and assume that the interaction described in the paper does not give rise to substantial spectral distortions. This requires the number of photons not to be conserved but rather evolve according to

$$\dot{n}_\gamma + 3Hn_\gamma = -\frac{3}{4}\Gamma\dot{\phi}n_\gamma , \quad (16)$$

where n_γ is the photon number density. In this case, the spectrum remains of blackbody form and therefore $\rho_\gamma = \pi^2 T^4/15$.

We have calculated the expected evolution of $T(z)$ (equivalent to the evolution of ρ_γ) as a function of redshift for various values of M . In Fig. 2, we display the numerical results for acceptable models and those which are in conflict with the data, together with the measurements. The prediction of General Relativity ($M \rightarrow \infty$), is also included. We subsequently find, through a chi-squared minimisation technique, that the *excluded* range (at 68% confidence level) for M is

$$1.53 \times 10^{-5} \text{ eV} \leq M \leq 3.07 \times 10^{-3} \text{ eV} . \quad (17)$$

This range of disallowed values of M contains the range of values for M for which Γ becomes infinite.

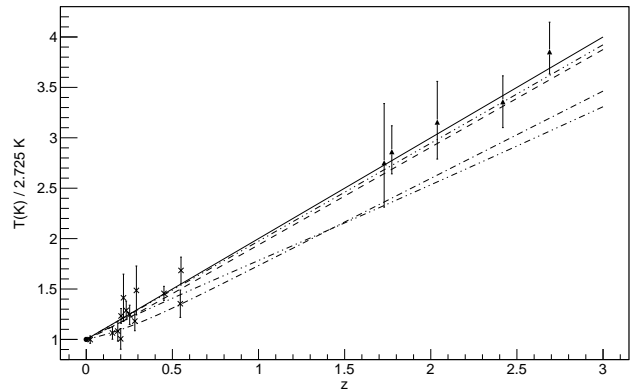


FIG. 2: Plot of the ratio $T(z)/T_0$ where $T_0 = 2.725\text{K}$, against redshift, z , for the exponential potential. The black circle marked at $z = 0$ is measured by COBE [15]. Different sets of measurements and their corresponding errors: Those marked with crosses are given by [17] and triangles are by [18]. There are five lines marked on this plot and each has an associated M (same as those in Fig 1): Solid line is $M \rightarrow 0, \infty$, dashed line $M = 1.53 \times 10^{-5} \text{ eV}$, dashed line with one dot $M = 2.17 \times 10^{-5} \text{ eV}$, dashed line with two dots $M = 3.10 \times 10^{-3} \text{ eV}$ and dashed line with three dots $M = 3.07 \times 10^{-3} \text{ eV}$.

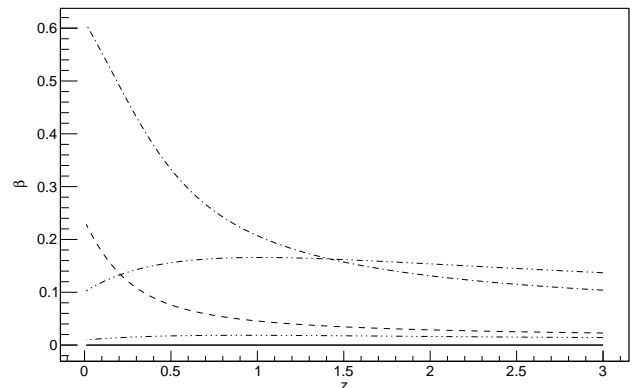


FIG. 3: For the exponential potential: The evolution of β with redshift, where β is defined by the expression $T(z) = T_0(1+z)^{1-\beta(z)}$. The various lines represent the universe evolving with different values of M : Solid line is $M \rightarrow 0, \infty$, dashed line is for $M = 1.53 \times 10^{-5} \text{ eV}$, the dashed line with one dot, $M = 2.17 \times 10^{-5} \text{ eV}$, the dashed line with two dots, $M = 3.10 \times 10^{-3} \text{ eV}$, and dashed line with three dots, $M = 3.07 \times 10^{-3} \text{ eV}$.

To quantify the observed deviation from General Relativity, we follow [17–19] and define a parameter, β , by $T(z) = T_0(1+z)^{1-\beta}$. We find that in the models discussed here, β is a function of z and can be rather large.

We find that β is given by the expression

$$\beta = \frac{1}{\ln(1+z)} \left\{ \ln \left(\frac{M^4 - \dot{\phi}^2}{M^4 - \dot{\phi}_0^2} \right)^{1/2} + \int_{\phi_0}^{\phi} \frac{\Gamma}{4} d\phi \right\}. \quad (18)$$

The evolution of this quantity is shown in Fig. 3. It is obvious to see that deviations of the evolution of $T(z)$ from General Relativity can be significant, making $T(z)$ a useful tool to constrain these couplings, and conveniently at relatively low redshift (in cosmological terms).

Let us now turn our attention to the inverse power law potential for the scalar field, (eq. (13)). The key distinction is a steeper slope, hence ϕ rolls faster for similar values of M . By the presence of $\dot{\phi}$ in eqs. (10) and (11), and indeed the metric, (eq. (3)), one can correctly deduce that a larger $\dot{\phi}$ will exacerbate the effects of the disformal coupling. The theoretical bounds on M that arise from Γ going singular are extended and shifted for an inverse power-law potential. This excluded range is

$$1.67 \times 10^{-5} \text{ eV} \lesssim M \lesssim 7.2 \times 10^{-3} \text{ eV}. \quad (19)$$

We in fact discover that this singularity range overlaps the excluded range set by the data. Γ goes singular for large M while still allowed by observation. We can then present only an experimental lower limit, again at 68% confidence and we arrive at and excluded range of

$$7.61 \times 10^{-6} \text{ eV} \leq M \lesssim 7.2 \times 10^{-3} \text{ eV}. \quad (20)$$

Note that the limit at large M is only dictated by the demand that Γ remains finite. The cosmology is sensitive to this limit. In our numerical analysis we find that for the inverse power law potential, the deviations of $T(z)$ from its value as predicted in General Relativity is becoming very large for large redshifts and therefore other observables, such as the redshift of recombination, will severely constrain M further. This requires further analysis, including a study of the evolution of perturbations in the coupled photon-baryon fluid which is beyond the scope of this paper.

To conclude, we have shown that without the presence of any conformal coupling, the CMB temperature constrains disformal couplings of radiation to a scalar field. In contrast, optical experiments on earth require the presence of some additional form of conformal coupling,

[10], to constrain disformal couplings. Our work can and should be extended in several ways: first, by generalising the coupling of ϕ to other matter forms. In this case, the expression of Q will look different and is most likely suppressed by ρ_{matter} . We expect that the range of M which is disfavoured by the data will change, and will no longer be able to set $C = 1$, while maintaining generality. This will mean taking conformal couplings into consideration, which unlike their disformal counterparts, require an additional screening mechanism to comply with sub-solar system experiments. This leads to the second item, that a disformal coupling scenario with a chameleon or symmetron mechanism must be investigated. In such theories, $\dot{\phi}$ is much smaller and we again expect this to have an influence on the excluded ranges for M . In addition, the creation of spectral distortions due to the disformal coupling has to be studied. Apart from possible distortions generated by the processes discussed in this paper (which would be the case if the adiabaticity condition in eq. (16) is violated), spectral distortions might also be generated when CMB photons travel through intergalactic magnetic fields, similar to what happens in the case of axions [20].

Nevertheless, we have shown that current measurement of the CMB temperature as a function of redshift provides information about any possible disformal couplings of radiation to scalar fields. Future observations will provide more stringent constraints by either becoming more accurate or by being extended to cover a larger range of redshifts. A study of the evolution of perturbations in the radiation field is also warranted, since the disformal coupling will also affect CMB anisotropies. We expect that in particular the inverse power-law potential will be severely constrained by these considerations.

Acknowledgments

The work of CvdB is supported by the Lancaster-Manchester-Sheffield Consortium for Fundamental Physics under STFC grant ST/J000418/1. SV is supported by an STFC doctoral fellowship. We thank Andrew Fowlie for useful discussions. We are grateful to Jens Chluba for important comments on possible spectral distortions of the blackbody spectrum in these theories.

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